Independence-friendly logic (IF-logic) and applications

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Abstract

Independence-friendly logic is a system of logic introduced by Hintikka and Sandu (1989). It extends ordinary first-order logic with arbitrary patterns of quantifiers and connectives, e.g.

$$\forall x \exists y (\exists z / \{x\}) R(x, y, z)$$

The original interpretation of IF formulae is in terms of games of imperfect information played by two players, Abelard and Eloise, the former choosing individuals from a domain of discourse to be the values of universally quantified variables and the latter doing the same for existentially quantified variables. The slash indicates the fact that when choosing a value for z Eloise does not know the value chosen by Abelard for x (but she "remembers" the value she chose earlier for y).

In addition to the game-theoretical interpretation, IF-logic has been given a compositional interpretation (Hodges, 1997) in terms of *teams* (originally called *trumps*), which motivated a reformulation of the syntax of IF-logic as what nowadays is known as *Dependence Logic* (Väänänen, 2007). In *Dependence Logic* one expresses dependencies (and independencies) by atomic formulae. Thus the IF formula above is rendered by

$$\forall x \exists y \exists z (Dep(y, z) \wedge R(x, y, z))$$

where the dependence atom $Dep(y, z)$ "says" that z depends (only) on y. Recently *Dependence Logic* has found applications to Data Base theory, quantum theory and inquisitive semantics. Hintikka (1996) has argued for the significance of IF-logic for the foundations of mathematics and language theory.

In my presentation I will describe some of the main concepts of the two approaches, the motivation behind them and evaluate their significance from a foundational point of view.